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THE OPERATION OF MAGNETIC AMPLIFIERS WITH VARIOUS TYPES OF LOADS

PART II - CONTROLLING THE ANGLE OF FIRING

THE TRANSFER CHARACTERISTICS OF AMPLIFIERS WITH LOW CONTROL IMPEDANCE

L. A. FINZI AND R. R. JACKSON

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Introduction

The companion paper Part I (Ref. 1) has supplied expressions for output currents and for currents in the load in terms of the angle of firing α taken as the independent variable. The treatment has been developed with reference to practical situations of magnetic amplifier circuitry, but substantially no need was felt for a close understanding of the phenomena taking place in the amplifier proper.

To obtain transfer characteristics relating load currents to amplifier signal a further step must be taken; namely one must determine the signal required to set a wanted angle of firing. It is under discussion in the art whether the output of a magnetic amplifier is "controlled" truly by magnetomotive forces or by voltages or, conceivably, by some other variable. One may say that this is a matter of personal taste, as any complete analysis of these electrically and magnetically interlinked systems requires consideration of both Faraday's law and Ampere's circuital law. But certainly for certain types of amplifiers (Ref. 2) the analysis is attacked profitably in terms of voltage equations only; no need is felt (up to a certain point) to consider the magnetomotive force equations of the cores; hence control voltages (or volt-time areas) are taken as the primary factor governing the angle of firing and thus the output. As the analysis of Part I supplies information concerning the angle at which a core ceases being saturated the evaluation of the signal voltage required for any wanted "reset" is almost immediate in amplifier circuits of this kind.

On the other hand, the operation of two-core amplifiers of the kind shown in Fig. 8, Part I, is defined best in terms of the magnetomotive force of average control current, which is taken as the controlling signal. In the following the question of signal requirements for wanted angles of firing is investigated for conventional amplifiers of this second kind, in situations of "low" control circuit impedance.

As the attention is focused on the m.m.f. equations of the cores, proper knowledge of the physical relations between core fluxes and m.m.f.'s in static, as well as in dynamic, conditions is needed throughout. More or less accurate approximations are allowable for this, depending on the order of magnitude of the ampere-turn gain of the device.

1) The Transfer Characteristic of the Simple Saturable Reactor

This elementary device is obtained from the amplifier circuit of Fig. 8, Ref. 1, by elimination of the feedback turns N_f and associated bridge. In this case it is acceptable practically to approximate the core magnetization curve by a vertical segment through the origin and two horizontal straight lines. As long as a core flux ϕ is swinging between positive and negative saturation $\pm\phi_{sat}$, its magnetomotive force ℓH is very small and is assumed to be zero; magnetomotive forces other than zero yield positive or negative saturation.

With the symbols and sign conventions shown in that Figure, the magnetomotive forces of the two cores are

1-1) $\ell H_I = N_c i_c + N_g i_g$ and 1-2) $\ell H_{II} = N_c i_c - N_g i_g$

During prefiring intervals the voltage of the a.c. power supply is balanced by the rates of change of both core fluxes; $\ell H_I = \ell H_{II} = 0$, and thus $i_c = i_g = 0$. Whenever one of the cores reaches saturation firing occurs; the other core, unsaturated, acts as a current transformer while the voltage of the a.c. power supply is balanced by the load, in series with the resistances, inductances and voltage sources of the control circuit reflected through the coupling provided by the unsaturated core. Hence at any time in the cycle $N_c i_c = N_g |i_g|$. (Readers not familiar with this matter may gather quick orientation from Ref. 3.)

This "d.c. transformer law" relating control currents to gate winding currents is valid, regardless of the type of load, of the impedance of the control circuit and of whatever residual inductance may be found in windings of a core beyond its sharply defined saturation flux level. Transfer characteristics relating half-cyclic averages of amplifier output m.m.f. $N_g I_g$ to averages of control m.m.f. $N_c I_c$ are represented by portions of straight lines at 45° through the origin, up to breaking points which correspond to the driving of the amplifier into conditions in which both cores become saturated simultaneously at some time in the cycle. Hence, for the loads considered in Section 3, Part I, the half-cyclic averages of load current, $I_L = I_g$ and of control current I_c are related linearly by $I_L = N_c/N_g\, I_c$ over the region $\alpha < \beta < \alpha + \pi$.

For the rectified loads considered in Section 4, Part I, control and gate winding currents still are related by this law, but the load current coincides with the gate winding current only during forcing intervals. As relaxation initiates, i_g drops quickly to zero, while the load current i_L keeps flowing through relaxation paths. That is $I_c\, N_c/N_g = I_G = I'_{LS}$, but the total half-cyclic load current $I_{LS} = I'_{LS} + I''_{LS}$ receives a further contribution I''_{LS} from the relaxation interval. To calculate load currents for given α one must know whether the flow of current is discontinuous or continuous, as discussed in Sections 2 and 4 of Part I. Namely, one assumes discontinuous flow, to start with, and evaluates δ from eq. 2-5) and eq. 2-6). If $\delta < \alpha + \pi$ this assumption was correct and I'_{LS} and I''_{LS} are twice the values given by formulas 2-8) and 2-9). If δ is greater than $\alpha + \pi$ the assumption of discontinuous flow was incorrect; then I'_{LS} and I''_{LS} are given by formulas 4-4) and 4-5). (It should be noted that the eventuality of discontinuous flow never arises if E_L is zero or negative.)

The transfer characteristic, relating load currents to signal, is obtained by plotting $I_{LS} = I'_{LS} + I''_{LS}$ versus $I_c = N_g/N_c \; I'_{LS}$, and, with the occurrence of relaxation, ceases to be a straight line, as shown also by others (Ref. 4 and 5).

Moderate extensions of this analysis can be made when the saturable reactor circuit is modified by the addition of feedback turns N_f shown in Fig. 8a, Part I. The m.m.f. equations of the two cores become

1-3) $\ell H_I = N_c i_c + N_g i_g + N_f i_f$; 1-4) $\ell H_{II} = N_c i_c - N_g i_g + N_f i'_f$

Under the assumption $i_f = |i_g|$ (i.e. perfect rectifiers and no bridge "overlap" phenomena) these equations show that the d.c. transformer law modifies into $N_c i_c = N_g |i_g| \, (1 \mp N_f/N_g)$, where the sign - or + applies, depending on whether the m.m.f. of feedback and of signal are concordant or discordant. Transfer characteristics relating half-cyclic averages of signal and gate winding currents remain straight lines (properly tilted) regardless of the type of load. If relaxation occurs in the load, half-cyclic averages of load currents I_{LS} are calculated as before. Transfer characteristics are obtained by plotting $I_{LS} = I'_{LS} + I''_{LS}$ versus $I_c = N_g/N_c \; (1 \mp N_c/N_g) \; I'_{LS}$.

However, this overly simplified analysis loses practical validity as the number of feedback turns N_f is increased, and becomes absurd when $N_f = N_g$. This inadequacy is disturbing because conventional self-saturating amplifiers of the doubler, center tap or bridge types -- most common in the manufacturing practice -- behave exactly as external feedback amplifiers with $N_f = N_g$. A proof of this equivalence results from simple algebraic transformations of the corresponding circuit equations; (see Ref. 6).

Attempts to improve on the analysis above by introducing a finite slope for the unsaturated portion of the magnetic characteristic are only moderately useful and inherently artificial. A much more realistic and valuable approach has been chosen by Storm (Ref. 7) by discarding single-valued representations and by expressing the magnetization curve in the unsaturated region by a rectangle with vertical sides. This dynamic hysteresis loop yields the magnetomotive forces of the amplifier cores (and thus the instantaneous control currents) during the pre-firing interval of swinging fluxes.

But no information is expressed in this way on the m.m.f. requirement of the unsaturated core during the remaining part of the half-cycle. If the static hysteresis loop is very narrow its m.m.f. during this part can be assumed to be zero. But this simplification is less adequate for common cores (i.e. 0.002" Deltamax, Hypernik V, etc.) as pointed out by Lord in the Discussion of Storm's paper and in Reference 8. In fact during the output interval, while the one core is definitely saturated, the other core, unsaturated, wanders slowly along the bottom of the minor loop from the left toward the right side, and the instantaneous control current is not zero nor does it average zero over this interval. The time-varying magnetomotive force of the unsaturated core is governed by a variety of complex phenomena presently understood only in part. Further analytical attempts must be preceded by a careful and discriminating observation of such elusive effects.

The semi-quantitative considerations that follow are based on experimental investigations of this kind, jointly with circuit analysis, to remove some of the vagueness that affects the considerable empirical corrections (or "shearing" factors) commonly needed to match analytical predictions to actual experimental transfer characteristics. Particular attention is given to the "positive" (high-gain) branch of the transfer characteristic. Purely resistive loads are considered first; then extensions are made to more general cases.

2) The Transfer Characteristics of the Doubler with Resistive Loads

The amplifier circuit of Fig. 8-b, Part I, is considered, with load R_L, purely resistive. The m.m.f. equations, with the symbols and sign conventions shown, are

2-1) $\ell H_I = N_c i_c + N_g i_{gI}$ and 2-2) $\ell H_{II} = N_c i_c + N_g i_{gII}$

and the voltage equations are, for the gate loops

2-3) $v_g = V_{gm} \sin \omega t = N_g \dfrac{d\phi_I}{dt} + \rho_g i_{gI} + R_L i_G + v_{rI}$, where $i_G = i_{gI} - i_{gII}$

2-4) $v_g = V_{gm} \sin \omega t = -N_g \dfrac{d\phi_{II}}{dt} - \rho_g i_{gII} + R_L i_G - v_{rII}$,

where ρ_g is the ohmic resistance of one gate winding and v_{rI} and v_{rII} are the voltages across the rectifiers, (positive drops for the chosen positive direction of currents),

and 2-5) $E_c = R_c i_c + N_c \dfrac{d\phi_I}{dt} + N_c \dfrac{d\phi_{II}}{dt}$ for the control loop

where E_c is the d.c. voltage applied, R_c is the total resistance of the control circuit.

Defining 2-6) $i_c = I_c + i_{ch}$ (where $I_c = E_c/R_c$ and i_{ch} is the harmonic content of the control current, averaging zero over the half-cycle) eq. 2-5) transforms into

2-7) $0 = R_c i_{ch} + N_c \dfrac{d\phi_I}{dt} + N_c \dfrac{d\phi_{II}}{dt}$

This algebraic transformation corresponds to a substitution of the actual control circuit with <u>two</u> separate circuits, both with N_c turns on each core. One circuit is supplied from a <u>constant current source</u> I_c, (where $I_c = E_c/R_c$ by definition); the other, with total resistance R_c, has no applied voltage, i.e. is short circuited.

By subtraction of 2-4) from 2-3)

2-8) $$0 = N_g \frac{d\phi_I}{dt} + N_g \frac{d\phi_{II}}{dt} + r_g i_{gI} + r_g i_{gII} + v_{rI} + v_{rII}$$

which can also be written directly as the voltage equation of the external loop of the two gate windings in series. Using 2-7), eq. 2-8) transforms into

2-9) $$0 = r_g i_{gI} + r_g i_{gII} + v_{rI} + v_{rII} - R_c i_{ch} N_g/N_c$$

This equation is useful as it gives some clue as to which rectifier is conducting at any time.

It is assumed at first that the rectifiers have no appreciable leakage. (That is, leakage, if any, shall be considered later by way of corrections, see Section 7.) Rectifiers satisfying this condition are readily available to the experimenter. Also it is assumed that the two cores have equal sizes and magnetic properties as should be the case if proper core matching techniques are used in amplifier production.

The customary assumption $R_c = 0$ shall be avoided here as it may give rise to misunderstandings about its meaning. Rather, in the joint experimental and analytical study that follows, cases of "low" control circuit resistance and cases in which this resistance is increased, e.g. by the addition of sizable forcing resistors, shall be considered in succession.

The operation of a doubler with purely resistive loads is evidenced by the oscillograms of Fig. 1. (Core data: Hypernik V, ID $1\frac{1}{2}$", OD $2\frac{1}{2}$", height $\frac{1}{2}$", thickness of laminations 0.002".) In this case R_c (or more properly R_c/N_c) is "low", as the control winding on each core occupies about one-half the available winding space, while the control voltage source E_c has negligible internal resistance. For this amplifier the rectifiers are Germanium AJA1A3, G. E. Co., $N_c = N_g = 2500$ turns, $R_c = 2 \times 65$ ohms, $r_g = 75$ ohms. The load resistance is $R_L = 400$ ohms in the tests of Fig. 1; moreover, the crest V_{gm} of the 60-cycle gate supply voltage is $V_{gm} = \sqrt{2} \times 130$ volts; this value is only slightly lower than the crest $V_{gN} = \omega N_g \Phi_{sat}$ of the "normal excitation" voltage, swinging the core fluxes from positive to negative flux saturation level $\pm \Phi_{sat}$ and vice versa.

Fig. 1-a shows the major dynamic loop of one core at the operating 60-cycle frequency and a minor loop described in the operation at a rather early angle of firing over the "positive" range of the transfer characteristic; Fig. 1-b and Fig. 1-c show instantaneous values of i_c and i_{gI} (at the same scale) during one positive half-cycle (during this half-cycle no detectable i_{gII} is recorded).

<u>Interval 1.</u> At $\omega t = 0$, core I is found at some initial ("pre-set") flux level Φ_p, while core II is at a level somewhere on the top of the loop near the "residual" level. During the pre-firing interval Φ_I climbs toward $+\Phi_{sat}$ while Φ_{II} diminishes at a nearly equal and opposite rate, as shown by eq. 1-7). Both fluxes are varying co-sinusoidally in time (or very nearly so) and their

magnetomotive force requirements result from portions of a minor loop which in its descending branch is seen to be very nearly coincident with the major <u>dynamic</u> loop embodying effects of eddy currents and domain boundary movements.

During this interval i_{gII} cannot be negative in the absence of rectifier leakage. It is shown here that it cannot be positive either. In fact, $\Lambda_g i_{gI}$ and v_{rI} are certainly positive, hence eq. 1-4) indicates that the sum ($\Lambda_g i_{gII} + v_{rII} - R_c i_{ch} N_g/N_c$) must be negative. Now, over the positive branch of the transfer characteristic $|i_c| > |I_c|$ in the pre-firing interval from 0 to α (as I_c is obtained from an averaging process of i_c over the whole half-cycle). That is, with our sign conventions, i_{ch} is a negative quantity during pre-firing and thus $(- R_c\, i_{ch}\, N_g/N_c)$ is a positive number. Since i_{gII} cannot be negative, it follows that v_{rII} <u>must</u> be negative; i.e. a reverse voltage appears across the gate rectifier II and therefore i_{gII} cannot be positive either.

(On the basis of superficial thinking one could be inclined to take for granted that no positive i_{gII} can flow during the half-cycle considered. Such "intuitive" reasoning will be shown to be fallacious as <u>positive</u> i_{gII} may well flow in later portions of the half-cycle considered.)

With i_{gII} 0, eq. 1-2) yields $i_c = \ell H_{II}/N_c$ where ℓH_{II} is obtained from the descending branch of the major dynamic loop.

Analytically this loop can be approximated by a parallelogram of width $2\,\ell H_d$ with slopes $d\Phi/d\ell H = \Phi_{sat}/\nu_d \ell H_d =$ constant. As eq. 1-4) yields

$$\Phi_{II} = \Phi_{sat} - (1-\cos\omega t) V_{gm}/N_g\omega ,$$

2-10) $$\ell H_{II} = -\ell H_d \left\{ 1 - \nu_d + \nu_d \frac{V_{gm}}{V_{gN}} - \nu_d \frac{V_{gm}}{V_{gN}} \cos\omega t \right\}$$

where $$V_{gN} = \omega N_g \Phi_{sat}$$

Hence the contribution I_c given by this first pre-fixing interval to the total I_c is

2-11) $$I_c' = \frac{1}{\pi}\int_0^{\alpha} i_c\, d\omega t = \frac{1}{\pi N_c}\int_0^{\alpha} \ell H_{II}\, d\omega t = -\frac{\ell H_d}{\pi N_c}\left\{ \left(1-\nu_d+\nu_d\frac{V_{gm}}{V_{gN}}\right)\alpha - \nu_d \frac{V_{gm}}{V_{gN}} \sin\alpha \right\}$$

which for $V_{gm} \cong V_{gN}$ reduces to

2-12) $$I_c' = -\frac{\ell H_d}{\pi N_c}\left(\alpha - \nu_d \sin\alpha\right)$$

The simplified eq. 2-12) coincides with an expression derived by Phillips in the Discussion of Reference 6, but it is only a part of the total I_c. The assumption $V_{gm} \cong V_{gN}$ is accepted throughout the following text.

Interval 2. At $\omega t = \alpha$ the ascending Φ_I reaches $+\Phi_{sat}$ and from there on $d\Phi_I/dt$ takes an entirely different (and much smaller) order of magnitude; according to eq. 2-3) i_{gI} is now limited primarily by the load and gate winding resistances. The phenomena taking place during the firing transition are not easily described; the firing shock may cause some oscillations in the various loops with spikes and dips in i_c. Fortunately these phenomena are short-lived and may be ignored in an attempt to evaluate I_c by an averaging integration extended over the whole half-cycle.

If firing occurs at $\alpha < \pi/2$, i_{gI}, after its firing jump, keeps increasing, reaching its peak $i_{gI\ peak} = V_{gm}/R$ at $\alpha \cong \pi/2$. Accordingly Φ_I at that time reaches its maximum value Φ_{max}, where $\Phi_{max} > +\Phi_{sat}$, since the "saturated" incremental permeabilities of core I, small as they appear, are certainly not zero.

Eq. 2-7) indicates that Φ_{II} keeps decreasing during this second time interval (this is generally true because i_{ch} is positive over this interval for all but very small values of α; while also $R_c\, i_{ch}$ is generally small in comparison to $N_c\ d\Phi_{II}/dt$ over this interval). But both flux changes are now slower, by an entirely different order of magnitude. Therefore the m.m.f. requirements of the still descending core II are no longer expressed by its dynamic major loop and are given more nearly by its static major loop, as recognized in Ref. 7. This loop can be approximated by a parallelogram of width $2lH_s$ and of slope $d\Phi/dlH = \Phi_{sat}/\gamma_s lH_s$ where $\gamma_s = (OD-ID)/(OD+ID)$; (see Ref. 9).

Considering that the downward excursion on the static loop is very limited, it appears that i_c over this interval (in which i_{gII} is still zero) is expressed by

$$i_c = -\chi \frac{lH_d}{N_c}\left(1-\gamma_s\cos\alpha\right) = \text{constant}, \quad \text{where } \chi = 2lH_s/2lH_d$$

The contribution I_c'' given to the total I_c by this interval between α and π is

$$I_c'' = -\frac{\chi l H_d}{\pi N_c}\left(1-\gamma_s\cos\alpha\right)\left(\pi/2-\alpha\right) \qquad \text{valid for } \alpha < \pi/2. \tag{2-13}$$

No such contribution appears in the over-all expression of I_c, if firing occurs at $\alpha \geq \pi/2$, that is

$$I_c'' = 0 \qquad \text{if } \alpha \geq \pi/2 \tag{2-14}$$

Interval 3. In the remaining part of the half-cycle i_{gI} decreases and and accordingly Φ_I also decreases from Φ_{max}. Hence a process begins in which, with $R_c\, i_{ch}$ small in comparison to $|N_c\ d\Phi_{II}/dt|$, the flux Φ_{II} starts increasing (very slowly) toward the level $\Phi_{II} = \Phi_p$ at the end of the half-cycle. During this third time interval the magnetomotive force requirements of the unsaturated core II wandering from the left to the right side of the static loop are somewhat obscure.

Attempts to give analytical expressions to the incremental permeabilities corresponding to this portion of the bottom of the minor loop have not been satisfactory so far, as any approximation tried does not explain some intriguing experimental evidence; for instance, the oscillographic trace of ℓH_{II} as a function of time over this third interval seems to be affected only slightly by changes of R_L or V_{gm} (and thus of $i_{g\,peak}$) or of R_c within somewhat broad ranges. (This is a matter for further investigation and analyses presently in course.)

But the very invariance of this trace, recognized from experimentation performed on various amplifiers for different cores, allows for an empirical approximation of i_c over this third interval. Namely, with $i_{gII} = 0$ (as is still well the case for "low" values of R_c) the parabolic approximations are suggested

2-15) $$i_c = -\frac{\chi \ell H_d}{N_c}\left[1 - V_s \cos\alpha - 8\left(\frac{\omega t - \pi/2}{\pi}\right)^2\right] \qquad \text{if} \quad \alpha < \pi/2$$

and 2-16) $$i_c = -\frac{\chi \ell H_d}{N_c}\left[1 - V_s \cos\alpha - 2\left(\frac{\omega t - \alpha}{\pi - \alpha}\right)^2\right] \qquad \text{if} \quad \alpha \geq \pi/2$$

Hence the contribution given by i_c over this third interval is

2-17) $$I_c''' = -\frac{\chi \ell H_d}{\pi N_c}\left[1/3 - V_s \cos\alpha\right](\pi - \pi/2) \qquad \text{if} \quad \alpha < \pi/2$$

or 2-18) $$I_c''' = -\frac{\chi \ell H_d}{\pi N_c}\left[1/3 - V_s \cos\alpha\right](\pi - \alpha) \qquad \text{if} \quad \alpha \geq \pi/2$$

The total control current I_c needed for any wanted angle of firing is thus

2-19) $$I_c = I_c' + I_c'' + I_c''' = \text{function of } \alpha$$

where I_c' is given by eq. 2-12) while I_c'' is given by eq. 2-13) or eq. 2-14) and I_c''' by eq. 2-17) or eq. 2-18) depending on whether $\alpha < \pi/2$ or $\alpha \geq \pi/2$. (Oscillogram 1-d evidences the immediate transition from interval 1 to interval 3 for a case of $\alpha > \pi/2$.)

On the other hand, the load current for given α is obtained from the type of reasoning used in Part I, with some correction for contributions given by pre-firing gate currents, whenever significant. That is,

2-20) $$I_L = I_G = I_{gIs} + I_{gIp} = \text{function of } \alpha \text{, where}$$

2-21) $$I_{gIS} = \frac{1}{\pi}\int_{\alpha}^{\pi} i_{gI}\,d\omega t = \frac{V_{gm}}{\pi R}\left(1+\cos\alpha\right),\ (\text{with } R = R_L + r_g)$$ is

the contribution given to the half-cyclic average of gate current by the most significant saturation (post-firing) interval, and

2-22) $$I_{gIP} = \frac{1}{\pi}\int_{0}^{\alpha} i_{gI}\,d\omega t \cong 2\frac{\ell H_d}{N_g}\frac{\alpha}{\pi}$$ is the contribution of the magnetization (pre-firing) part of the half-cycle. Eq. 2-22) is based on the coarse assumption that the m.m.f. requirement of core I climbing toward saturation is obtained from $\ell H_I = N_c i_c + N_g i_g$ with $\ell H_I = +\ell H_d$ throughout. This approximation yields most nearly correct results for $\alpha \cong \pi$ where I_{gIP} may be most significant, while its significance becomes per cent-wise smaller as α approaches zero.

Under elimination of α from the two parametric equations 2-19) and 2-20) the transfer characteristic $I_L = f(I_c)$ is obtained. Also an analytical expression is obtainable for the incremental ampere-turn gain K_{AT}.

2-23) $$K_{AT} = \frac{dI_L N_g}{dI_c N_c} = \frac{N_g}{N_c}\,\frac{dI_L/d\alpha}{dI_c/d\alpha} = \text{function of } \alpha.$$

Curve (a) in Fig. 2 shows an experimental transfer characteristic for the amplifier and load described. The same curve is repeated on the right and compared with a calculated one. (It is noted that $dI_c/d\alpha$ takes two slightly different values at $\alpha = \pi/2$, depending on whether formulas 2-13 and 2-17 or 2-14 and 2-18 are used, approaching $\pi/2$ from $\alpha < \pi/2$ or from $\alpha > \pi/2$. This discontinuity of incremental gains which is not detected in experimental characteristics, results from the simplified analytical fit chosen to describe the interval 3. More elaborate formulation would eliminate this flaw, however at the expense of the numerical calculations used to describe the transfer characteristic. On the whole, calculated and experimental incremental gains appear to agree rather well over the whole range of operation.)

It should be clear at this point that the formulas suggested are no better than clumsy attempts to coordinate analytically certain phenomena. The considerations given may be of some value in so far as phenomena are pointed out which should be properly considered and weighed by designers searching for "formulas" of their own.

The relative significance of the constants H_d, H_s, V_d and V_s is recognized by inspection of the formulas. This may be of interest in the question of pertinent core testing procedures. It may be noted also that the operation described is not limited to cases of finite positive slopes of the major loop. In fact, $d\Phi/dH$ could be infinite or even negative (as mentioned by Phillips) and no immediate reason could be seen in this to decide upon instabilities in the operation.

Modified expressions for I_c may be needed, for instance, whenever the dynamic loop cannot be approximated completely by a parallelogram as it presents bulges, e.g. at the beginning or end of the descending branch. No attempts have been made to embody in the formulas "triggering" effects, recognized generally at the lower end of the characteristics and discussed in Ref. 10.

Influence of forcing resistors. The assumption of "very low" control circuit resistance can be removed now examining how the control current i_c is modified during the three intervals described when R_c is increased within certain limits by means of forcing resistors added in the control circuit. Eq. 2-7) shows now that the rates of change of Φ_I and Φ_{II} no longer can be considered as equal and opposite, though for practical values of the forcing resistor their orders of magnitude in the various intervals may not be affected substantially.

During the pre-firing interval i_{ch} is mostly negative, then $|d\Phi_I/dt| < \frac{d\Phi_1}{dt}$, thus $v_{rII} < 0$ and $i_{gII} = 0$. Φ_I descends at a rate which has been slowed down by the presence of the forcing resistor; but, as long as $R_c\, i_{ch}\, N_g/N_c$ is small in comparison to v_g, this rate of change remains of an order of magnitude that justifies the use of the major dynamic loop to obtain i_c (and thus I_c') during this interval.

Again, at $\omega t = \alpha$, i_{gI} jumps and, if $\alpha < \pi/2$, it keeps increasing further until it reaches its maximum at $\omega t = \pi/2$ or very nearly so. During this interval, as Φ_I keeps increasing slowly, Φ_{II} decreases also slowly and the static loop supplies i_c. No general statements can be made about the sign of i_{ch} and therefore about the influence of the voltage $R_c\, i_{ch}$ upon the rate of change of Φ_{II} during this interval. For angles of firing already a little larger than 0, i_{ch} is generally slightly positive; then the forcing resistor increases somewhat the rate of change of the very slowly descending Φ_{II}, and in fact Φ_{II} may keep descending along the static loop even while i_{gI}, decreasing from its peak, causes Φ_I to recede from Φ_{max}. In this case the second interval is protracted beyond $\pi/2$ and this results ultimately in a somewhat larger value of I_c for given α.

Other more striking effects, which take place in the doubler, generally toward the end of the voltage half-cycle, are more largely responsible for the overall increase of I_c due to the forcing resistors. In fact, as core II is wandering from the left to the right side of the static loop the unexplained wave form of lH_I appears substantially unaffected by the introduction of the forcing resistor. But already <u>before</u> the reversal of line voltage at $\omega t = \pi$ a <u>positive</u> flow of i_{gII} (with the sign conventions of Fig. 8-b, Part I) can be observed. In fact, eq. 2-9) with $r_g\, i_{gI}$ approaching zero and with i_{ch} certainly positive over this region, indicates the possibility that v_{rII} may no longer be negative.

This is an "overlap" phenomenon of simultaneous conduction of both rectifiers, which is well evidenced in Fig. 2. As i_{gII} now gives a (positive)contribution to lH_I, i_c is modified. An exact analysis of the overlap does not seem easy for a number of obvious reasons; all the various terms appearing in eq. 2-9) are of the same order of magnitude (and very small) as the end of the half-cycle nears; departures of i_{gI} from the sinusoidal wave forms are noticed; threshold voltages of both rectifiers are significant and, moreover, while one recognizes that i_{ch} is positive now, no quantitative expressions for it are available. Qualitatively one sees that overlap initiates earlier if $R_c N_g/N_c$ is large in comparison to the gate winding resistance R_g, and if the threshold voltages of the rectifiers are low.

The transfer characteristic (b) in Fig. 2 evidences the increase of control current requirements due to a sizable increase of R_c. On the other hand, curve (e) indicates the effect of total "constraint" when i_{ch} is made zero by the use of a 4500 henry reactor (3.4 Megohms for the second harmonic of the 60 cycle power supply frequency) in series in the control circuit.

Finally, curve (g) shows the characteristic obtained multiplying by two the output of a half-wave amplifier with constant control current, $i_c = I_c$. The preceding description of the phenomena taking place in the unconstrained doubler indicated already that hasty generalizations of constrained half-wave amplifier behavior to unconstrained doublers, popular as they may be, are not warranted, unless one is satisfied with the rather obvious recognition that in either case signal currents somehow control the amplifier between the two limits of full and minimum output.

It should be added that additional magnetomotive forces introduced by means of bias windings simply translate horizontally the characteristic (a). On the other hand, the characteristic obtained in (b) is transformed into (a), and then translated horizontally, if the described effects of the comparatively high constraining R_c used to obtain (b) are eliminated by the parallel action of some <u>low resistance</u> bias circuit. More precisely, in the case of many control or bias circuits excited by d.c. voltages, the output is dictated by the algebraic sum of their average magnetomotive forces. The degree of "constraint" of the amplifier, and thus the effects of overlap etc., depend on the magnitude of the summation $\sum N^2/R$ extended to all control circuits. (This point has been investigated experimentally anew because doubts have been raised on this matter and hence on the legitimacy of the use of the concept of average control magnetomotive forces as "signals" in this kind of amplifiers.)

<u>The "negative" branch of the characteristic</u>. The considerations that precede apply over the so-called "<u>positive</u>" branch (i.e. the <u>high-gain</u> portion) of the transfer characteristic, in which the output is controlled from its maximum down to its minimum value by increases of the absolute value of the negative I_c. Minimum output is obtained when $\alpha = \pi$, that is, when $I_c = - \ell H_d/N_c$. In this particular condition no firing occurs, as both fluxes swing up and down without ever exceeding the levels $\pm \Phi_{sat}$, and the cyclic average of output current $I_L = I_G$ is given by $I_G = I_{g\pm p} \cong 2\frac{N_c}{N_g}|I_c|$

Further increases of $|I_c|$ cause the output to rise again, though with considerably <u>smaller ampere-turn gains</u>. To describe this branch of the characteristic, let it be assumed that at the beginning of the positive voltage half-cycle Φ_I is at some level near the <u>negative</u> residual flux and that Φ_{II} is at some level between the lower and upper limits of the major dynamic loop. At $\omega t \cong 0$ the usual process of swinging fluxes initiates as Φ_I climbs and Φ_{II} comes down. The m.m.f. requirements of the descending core may still be dictated by the major dynamic loop, but this may not be sufficient to define i_c in this case, because with $|I_c| > \ell H_d/N_c$, the harmonic term i_{ch} is positive and "overlap" (that is flow of positive i_{gII}) is observed frequently in the pre-firing interval. Higher values are taken by i_c after firing. In fact, firing occurs here when, at some $\omega t = \alpha$, the <u>descending</u> flux Φ_{II} reaches $-\Phi_{sat}$. From then on the a.c. source sees the load resistance, in series with the reflected control circuit, since core I is not saturated and acts as a current transformer. During the firing interval $i_c \triangleq -N_g/N_c\, i_{gI}$. In conclusion the over-all ampere-turn gain N_gI_G/N_cI_c, (which at $\alpha = \pi$ was seen to be equal to <u>two</u>) decays gradually to become very nearly unity when $|I_c|$ is increased to give maximum output ($\alpha = 0$) at the left hand of this branch. In view of the comparatively limited significance of this branch of the transfer characteristic a more detailed analysis is omitted in this Paper.

3) The "Center-tap" and "Bridge" Circuits with Resistive Load

Without writing the voltage and m.m.f. equations of these circuits, it can be stated that the phenomena described in the preceding Section 2 are recognized without appreciable differences in these new cases. In fact also, oscillograms of gate and control current are quite similar to those presented in Fig. 1, but for the presence of some small rectifier leakage. (Of course, the higher reverse rectifier voltages appearing during saturation intervals of these circuits require the use of differently rated rectifying elements.) Formulas 2-11) to 2-19) apply here too; separate corrections to account for leakage can be introduced subsequently, as mentioned in Section 7.

As R_c is increased by means of forcing resistors the first of the two effects mentioned in the third interval still appears, that is, i_c remains at the level dictated by the static loop beyond the point at which i_{gI} reaches its peak. The second effect ("overlap"), however, is much less conspicuous and hardly noticeable. In fact, the voltages to be overcome to initiate any earlier flow of positive i_{gII} are inherently higher in these two amplifiers. As overlap is more nearly negligible, it may be expected that the transfer characteristics of these circuits should be much less sensitive to moderate increases of R_c than was noticed in the doubler. This evidenced by a comparison of the experimental transfer characteristics (c) and (d) of Fig. 2. An entirely different situation arises, however, in the case of extreme constraint, as shown by curve (f) of Fig. 2.

4) The Doubler with Non-rectified Inductive Loads

This kind of load has been treated in Section 3, Part I. For any value of α the nomogram supplies the corresponding angle of extinction β . The interval 1 initiates here upon extinction of the preceding saturation interval. That is, at $\omega t = \beta - \pi$, Φ_I is found at the pre-set level Φ_ρ and starts climbing toward saturation, while Φ_{II} decreases at a nearly equal rate from its initial level, somewhere near Φ_{res}. The rates of change of these fluxes may be somewhat different than in the pre-firing interval of the situation described in Section 2, because a different portion of the sine wave of v_g is being absorbed by the cores. By way of approximation, however, it shall be assumed that the portion of minor loop described by the descending core over this interval still coincides with a portion of the major dynamic loop. Thus, for this interval, as $\Phi_I = \Phi_{sat} - [\cos(\pi-\beta) - \cos\omega t]\, V_{gm}/\omega N_g$ with the major dynamic loop approximated by a parallelogram, (and with $V_{gm} \doteq V_{gN}$) it is found

$$\text{4-1)} \quad I_c' = \frac{1}{\pi}\int_{\beta-\pi}^{\alpha} i_c\, d\omega t = -\frac{\ell H_d}{\pi N_c}\left\{\left(1-\nu_d[1+\cos\beta]\right)(\alpha-\beta+\pi) - \nu_d(\sin\alpha + \sin\beta)\right\}.$$

At $\omega t = \alpha$ firing takes place and i_{gI} starts increasing; thus in this second interval Φ_I increases from Φ_{sat} toward $\Phi_{max} > \Phi_{sat}$ and accordingly Φ_{II} comes down at a very slow rate so that the m.m.f. requirements of core II are more nearly expressed by the static major loop. The downward excursion on this loop is very limited, hence i_c is substantially constant during this process and is given by

$$\text{4-2)} \quad i_c = -\chi\frac{\ell H_d}{N_c}\left[1-\nu_s(1+\cos\alpha+\cos\beta)\right]$$

This second interval lasts until i_{gI} reaches its peak at $\omega t = \xi$, where ξ is evaluated from Part I, Appendix B, Fig. 10. Hence

$$\text{4-3)} \quad I_c'' = \frac{1}{\pi}\int_{\alpha}^{\xi} i_c\, d\omega t = -\chi\frac{\ell H_d}{\pi N_c}\left[1-\nu_s(1+\cos\alpha+\cos\beta)\right](\xi-\alpha)$$

Beyond ξ the current i_{gI} decreases from its peak, Φ_I decreases accordingly and the third interval takes place, during which Φ_{II} rises slowly while the core m.m.f. wanders from the left to the right side of the static major loop. Again the wave form of ℓH_{II} can be approximated by a parabola. Hence

$$\text{4-4)} \quad I_c''' = \frac{1}{\pi}\int_{\xi}^{\beta} i_c\, d\omega t = -\chi\frac{\ell H_d}{\pi N_c}\left[\tfrac{1}{3}-\nu_s(1+\cos\alpha+\cos\beta)\right](\beta-\xi).$$

It is noted that all three intervals are always present for this kind of load. Regardless of the value of α, the post-firing i_{gI} shows a gradual rising in the interval from α to ξ; this is the interval 2. (It is seen here that precise knowledge of ξ is not too essential as the transition from interval 2 to interval 3 is very gradual in terms of i_c.)

Figure 3 shows oscillograms of i_c and i_{gI} to different scales. Also transfer characteristics with $R_c = 180$ ohms are shown. Here $R_L = 584$ ohms, $L = 1.86$ henries, and a d.c. source with $K = +0.20$ is included within a rectifying bridge of its own, as shown in Fig. 4, Part I.

The considerations above apply for low values of R_c. If forcing resistors are used, both effects of extensions of the second interval and of overlap before reaching β are observed again, with resulting increases of the half-cyclic control current requirements for given output.

5) Rectified Loads with Discharging Paths

In these cases, treated in Sections 4 and 5 of Part I, the load current i_L coincides with the amplifier output current i_G during forcing intervals; the relaxation current i''_L does not flow through the gate windings and therefore does not affect the control requirements for given α.

If it is known that the flow of load current is discontinuous, ξ is obtained again from Part I, Appendix B, Fig. 10. In more usual cases of continuous flow, ξ is found from Part I, Appendix B, Fig. 11, in conjunction with Fig. 10.

After $\omega t = \xi$, i_{gI} (and ϕ_I) decreases; ϕ_{II} starts rising and a third interval initiates in which the m.m.f. of the unsaturated core wanders from the left toward the right side of the static loop. In the analysis of Part I, Sections 2 and 4, no particular effort was made to evaluate the decay of gate current past the angle $\gamma \cong \pi$ at which relaxation initiates and the load current becomes independent of the gate winding current. On the other hand, now it is seen that this third interval really terminates when the gate current extinguishes; because of some saturated inductance L_g of the gate winding this occurs at some angle $\eta > \pi$, and the current i_{gI} over this interval is defined by the differential equation 2-3) of Part I. The angle η is found from the curves of Fig. 4, in terms of the initial value i_π of gate current when relaxation initiates, and of $\theta_G = \tan^{-1}(\omega L_g / R_g)$ (i_π is obtained for Part I, eq. 2-5) for cases of discontinuous flow, and from Part I, eq. 4-7) for the more common continuous flow).

Under these considerations the term I'_c is obtained from integration of

$$i_c = -\frac{\ell H_d}{N_c}\left\{1 - \nu_d\left(1 + \cos\eta + \cos\omega t\right)\right\} \qquad \text{from } \eta - \pi \text{ to } \alpha.$$

Likewise I''_c is obtained from integration of

$$i_c = -\frac{\chi \ell H_d}{N_c}\left\{1 - \nu_s\left(1 + \cos\eta + \cos\alpha\right)\right\} \qquad \text{from } \alpha \text{ to } \xi.$$

Careful observation of oscillograms of the magnetomotive force of the unsaturated core shows that interval 3 consists of two sub-intervals, from ξ to π and from π to η, with a break in the slope at π. But, to obtain workable formulas, the whole course of interval 3 can be justifiably approximated be a single parabolic expression

$$i_c = -\frac{\chi \ell H_d}{N_c}\left\{1 - \nu_s\left(1 + \cos\eta + \cos\alpha\right) - 2\left(\frac{\omega t - \xi}{\eta - \xi}\right)^2\right\}, \qquad \text{valid from}$$

ξ to η.

(All this is a matter of refinements which cannot be carried out further, because L_g is not too well defined numerically, as the process of decaying i_{gr} extends over a region in which transitions from $+\Phi_{sat}$ to Φ_{res} are taking place in the saturated core.) Tentatively the following formulas are suggested

2-19) $$I_c = I_c' + I_c'' + I_c'''$$

5-1) $$I_c' = \frac{1}{\pi}\int_{\eta-\pi}^{\alpha} i_c\, d\omega t = -\frac{\ell H_d}{\pi N_c}\left\{(1-\nu_d[1+\cos\eta])(\alpha-\eta+\pi) - \nu_d(\sin\alpha+\sin\eta)\right\}$$

5-2) $$I_c'' = \frac{1}{\pi}\int_{\alpha}^{\xi} i_c\, d\omega t = -\frac{X\ell H_d}{\pi N_c}\left[1-\nu_s(1+\cos\alpha+\cos\eta)\right](\xi-\alpha)$$

5-3) $$I_c''' = \frac{1}{\pi}\int_{\xi}^{\eta} i_c\, d\omega t = -\frac{X\ell H_d}{\pi N_c}\left[\tfrac{1}{3}-\nu_s(1+\cos\alpha+\cos\eta)\right](\eta-\xi).$$

Oscillograms of gate and control currents are shown in Fig. 5 for the amplifier in doubler circuit , but with the entire load within a rectifying bridge. Here $R_c = 180$ ohms, $R_L = 800$ ohms, $L_L = 2.5$ henries and $K = +0.20$.

A matter of concern is seen in the oscillations observed in all circuits and, in particular, in i_c upon the firing shock for this kind of load. The formulas above have been derived under the assumption that the effects of these oscillations average to zero in the integration process.

Fig. 5 presents also a comparison of calculated and measured transfer characteristics for the conditions described. Discontinuous flow occurs for $\alpha > 60°$, as recognized with the procedures given in Part I, Sections 2 and 4, and reviewed in Section 1. I_L is the sum of the forcing term I_{Ls}' and of the relaxation term I_{Ls}'' given by eq. 4-4) and 4-5), Part I, in cases of continuous flow and by twice the values of eq. 2-8) and 2-9), Part I, for cases of discontinuous flow. On the other hand, $I_g = I_{Ls}' + I_{gIP}$ where I_{gIP} is the correction due to pre-firing currents in the gate winding; $I_{gIP} \triangleq \ell H_d[\alpha-(\eta-\pi)]/\pi N_g$. (In the case of discontinuous flow extinguishing at δ a similar correction can be made for I_L adding to it $\ell H_d[\alpha-(\delta-\pi)]/\pi N_g$ to account for the fact that pre-firing magnetizing currents flow in the load during the interval between extinction and firing.)

6) Self-saturating Circuits with Additional Degenerative Feedback

Note: The historical development of magnetic amplifier analyses, starting from the simple saturable reactor and successively adding "feedback" turns N_f, made it customary to say that the simple reactor has zero feedback and that the addition of e.g. $N_f = 0.8\ N_g$ turns converts it into an amplifier with 80% feedback. With this terminology conventional doublers and similar self-saturating circuits are amplifiers with 100% feedback.

The recognition that the simple saturable reactor with sensitive core materials has unity ampere-turn gain later led to some suggestion that it would be more proper to say that the saturable reactor inherently has a 100% feedback. The external feedback amplifiers with $N_f = N_g$ (and thus also the conventional self-saturating circuits) should be more properly considered as the "basic" amplifier. That is, an external feedback amplifier with $N_f = 0.8\ N_g$ has a degenerative feedback of 20%. Likewise, if $N_f = 1.03\ N_g$, this amplifier has a regenerative feedback of 3%.

Degenerative action is obtained in the self-saturating amplifiers by introducing magnetomotive forces proportional to the rectified amplifier output current and acting on each core with a negative contribution to its total m.m.f. . According to a suggestion by Krabbe (Ref. 11) this can be obtained without need for other rectifiers, by adding N_d turns on each core; these turns are cross-connected in series with the gate winding on the other core with proper polarities. The degenerative action is seen from the recognition that the m.m.f. equations 2-1) and 2-2) modify into

6-1) $\ell H_I = N_c i_c + N_g i_{gI} - N_d i_{gII}$ 6-2) $\ell H_{II} = N_c i_c + N_g i_{gII} - N_d i_{gI}$

The operation of the amplifier in terms of total requirements of instantaneous m.m.f.'s ℓH_I and ℓH_{II} remains unaffected (but, perhaps, for minor departures in the "third" interval in which the unsaturated core wanders from side to side of the static loop, if R_c is large). Considering the operation of core II during the half-cycle initiating at $\omega t = 0$ (or at $\omega t = \beta - \pi$ for the case of non-rectified, inductive-resistive loads) it appears now that the various m.m.f. requirements of the core are satisfied for any given angle of firing if the original instantaneous control current i_c is varied by the addition of a term $i_c^* = N_d/N_c\, i_{gI}$. That is, for a given α (and thus for a given output) the control current I_c is given by

6-3) $$I_c = I_c' + I_c'' + I_c''' + I_c^*$$

where, for each load situation, I_c', I_c'' and I_c''' are the contributions given to the total control current in the absence of degenerative turns by the three intervals considered in the previous sections, and $I_c^* = + N_d/N_c\, i_{gI}$. The summation of $I_c' + I_c'' + I_c'''$ was seen to be a negative number; the additional requirements of degenerative action introduce a positive term. Thus the transfer characteristic expressing I_G ($I_G \neq I_L$ for the loads considered in Section 5) is modified by a kind of "shearing" to the right, and positive values of control currents may well be needed over certain portions of the transfer characteristic.

More commonly, degenerative feedback is obtained by feeding the N_d turns on the two cores directly in series with a bridge rectifying the amplifier output current i_G substantially with the same results. (In either case the addition of the N_d turns modifies the voltage equations of the gate loops; this is a simple matter which may be disregarded if $N_d \ll N_g$ and which, in any case, is not within the scope of this paper (see Ref. 6 and 12).

As the degenerative action is increased by making N_d larger and larger, the terms I_c', I_c'' and I_c''' become more and more insignificant with respect to I_c^* and no need may be felt for an accurate consideration of the minor loops described by the cores. Then, of course, coarser representations of the $\phi = f(\ell H)$ relationship, (and, in particular, the single-valued representation with infinite permeability in the unsaturated region) are adequate, as used in Section 1.

7) Rectifier Effects

Improper rectifier behavior is often responsible for some departures from the results of the previous analysis. Effects of rectifier "capacitance" are most elusive, and elementary analyses of "equivalent" linear circuits are inadequate for many obvious reasons. It is suspected that rectifier capacitance emphasizes unfavorably the high frequency oscillations which are observed in all the various coupled circuits upon the firing shock, with asymmetrical current wave forms. Inadequate knowledge of both rectifier "capacitance" and magnetic non-linearities along the bottom of the minor loop prevent the authors from attempting an analysis of these effects at this time.

On the other hand, effects of rectifier "leakage" are more readily accounted for. In fact, in cases of low control circuit resistances the reverse voltages which appear across the rectifier of the unsaturated core during saturation intervals are known in terms of output currents and circuit resistances (e.g. $v_{rI} \cong -r_g i_{gI}$ for the doubler, or $v_{rII} \cong -2v_g + r_g i_{gI}$ for the center-tap circuit, etc.). If enough information is available to relate these voltages to the negative i_{gII} of leakage, its magnetomotive force can be introduced in the expression of ℓH_{II}, thus modifying i_c during the intervals in which significant leakage occurs. Evidently this negative m.m.f. has "degenerative" effects quite similar to those described in Section 6, above. In fact, if a linearization in terms of a fictitious rectifier "reverse resistance" is admissible, rectifier leakage corresponds to a true degenerative feedback. The equivalent "N_d", and thus means for leakage compensations by the addition of regenerative turns $N_r =$ "N_d", have been calculated in terms of all pertinent parameters in Ref. 12, with simple results that are well usable, in all cases but the most critical ones of extremely high ampere-turn gains.

8) Regenerative Feedback and Related Effects

Regenerative action is obtained in the self-saturating amplifiers by addition of N_r turns on each core; these turns are cross-connected in series with the gate winding N_g on the other core with proper polarities. (Namely, regenerative action results if the current i_{gI} through these additional turns introduces a positive contribution to the m.m.f. ℓH_{II} of the other core.) The regenerative action is seen from the recognition that the m.m.f. equations 2-1) and 2-2) modify into

8-1) $\ell H_I = N_c i_c + N_g i_{gI} + N_r i_{gII}$ 8-2) $\ell H_{II} = N_c i_c + N_g i_{gII} + N_r i_{gI}$

It appears that the unchanged m.m.f. requirements of the core II are satisfied for any given angle of firing if the original instantaneous control current i_c is varied by the addition of a term $i_c^{**} = -N_r/N_c \, i_{gI}$. That is, the control current I_c for given α is given by

8-3) $I_c = I_c' + I_c'' + I_c''' + I_c^{**}$ where I_c', I_c'' and I_c'''

are the contributions given to the total control current in the absence of regenerative action and $I_c^{**} = -N_r/N_c \, I_{gI}$. The summation $I_c' + I_c'' + I_c'''$ was seen to be a negative number; I_c^{**} is also negative. The regenerative action modifies the positive branch of the transfer characteristic expressing I_G with a kind of shearing to the left. The same effects are obtained if the N_r turns are fed directly by a bridge rectifying the amplifier output current i_G.

A similar effect is recognized in center-tap amplifiers with inductive loads (or in external feedback amplifiers with inductive loads connected inside of the N_f loop) whenever no paths are offered for load relaxation by the additional provision of discharging rectifiers. In these cases, mentioned in Section 6, Part I, load relaxation takes place through the amplifier windings. In the case of the center-tap circuit the load current i_L of relaxation intervals finds two parallel paths through both gate windings, that is $i_L'' = i_{gI} + i_{gII}$.

During relaxation the cores are absorbing the a.c. supply voltage and their fluxes swing up and down along the major dynamic loop; that is

$$\ell H_I = N_c i_c + N_g i_{gI} \cong +\ell H_d$$
$$\ell H_{II} = N_c i_c + N_g i_{gII} \cong -\ell H_d,$$

whence, by summation

$$i_c \cong -\frac{N_g}{N_c}\frac{i_L''}{2}$$

In the case of continuous flow of load current these conditions prevail throughout the pre-firing interval. If the resistances of these relaxation paths do not affect the decay of relaxation current, one may say tentatively that the control current for given α is now expressed by

$$I_c = -\frac{N_g}{2N_c} I_L'' + I_c'' + I_c'''$$

If the flow is discontinuous these conditions are limited to the interval from $\omega t = 0$ to $\omega t = \delta - \pi$, as there is a portion of the pre-firing interval in which $i_L'' = 0$ and thus $i_{g\text{II}} = 0$. In this case

$$I_c = -\frac{N_g}{2N_c}\frac{1}{\pi}\int_0^{\delta-\pi} i_L''\, d\omega t + \frac{1}{\pi N_c}\int_{\delta-\pi}^{\alpha} \ell H_{\text{II}}\, d\omega t + I_c'' + I_c''' \qquad \text{, where}$$

ℓH_{II} is obtained from the descending branch of the major dynamic loop over the time interval in question.

The discharge of the load current i_L'' through the gate windings has an action similar to the one noticed in the regenerative feedback in so far as it increases $|I_c|$ for given output; it differs from the true regenerative action, as increases of $|i_c|$ are noticed in the pre-firing rather than in the saturation interval.

It is well known that the transfer characteristics thus calculated are most likely to yield multivalued outputs for certain ranges of I_c. Some of these outputs correspond to conditions of unstable operation. The question of instability, as caused by these recognizable regenerative effects, may be approached more specifically in a subsequent Paper.

9) Conclusions

An analysis of load behavior upon firing has been presented in the companion Paper, Part I, for certain common types of loads. This Paper, Part II, investigates the corresponding signal requirements dealing specifically with two-core amplifiers of the "low control impedance" types. Amplifiers with low ampere-turn gains have been considered first. The recognition of the inadequacies of the extension of simplified analyses to the more critical situations of amplifiers with very high ampere-turn gains has brought about a closer examination of the minor hysteresis loops described by the cores in the operation. Tentative formulas have been presented for the control current requirements in the case of "low" control circuit resistance; the modifying influence of moderate forcing resistors in the control circuit has been examined and "overlap" and other related effects have been described.

The study has been carried out first for purely resistive loads, as this seemed necessary at the present state of the art. Extensions to more general load situations have been made subsequently under use of the results of Part I and transfer characteristics have been obtained for the various cases.

Finally situations of less extreme gains (as resulting e.g. from an additional degenerative feedback or from the use of inadequate rectifiers) have been discussed. Also, a starting point for the further study of instabilities due to over-regeneration has been established.

A by-product of this approach is the tentative recognition of those core properties which seem to be most significant in the control of amplifiers with low control impedance; this may bring some clarification in the matter of pertinent core testing procedures and in the study of the problem of drifts.

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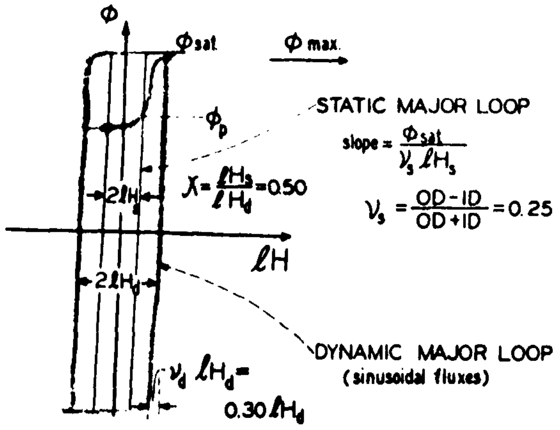


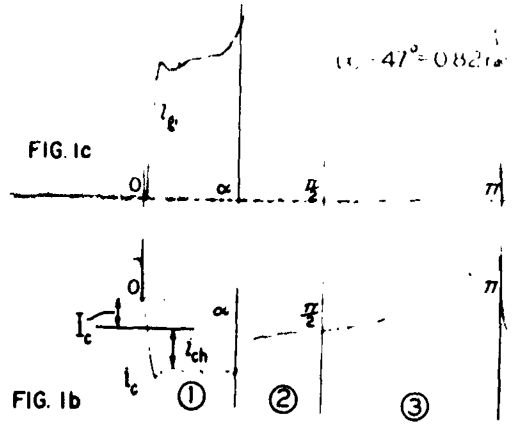
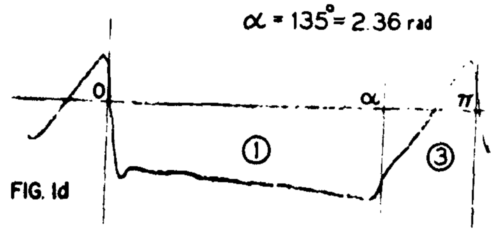


FIG. Ia

Fig. 1 Operation of the "doubler" with resistive load.



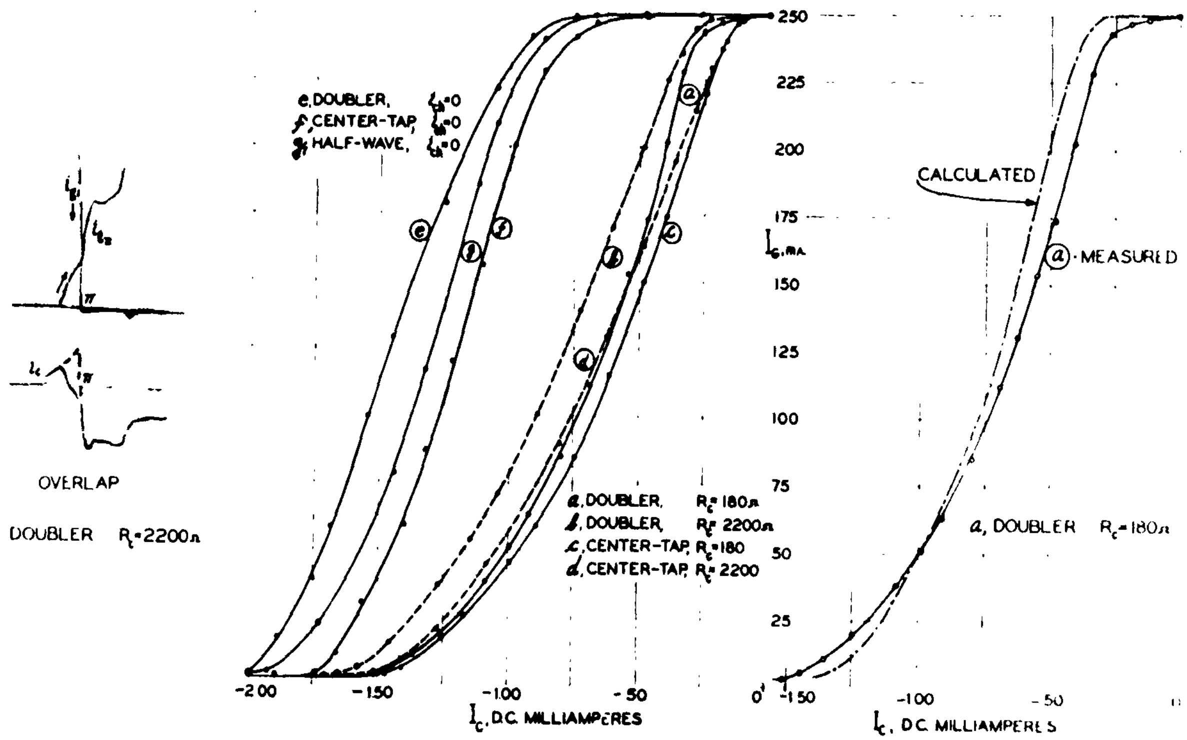


FIG. 2

Fig. 2 Transfer characteristics, resistive load. Influence of forcing control resistors.

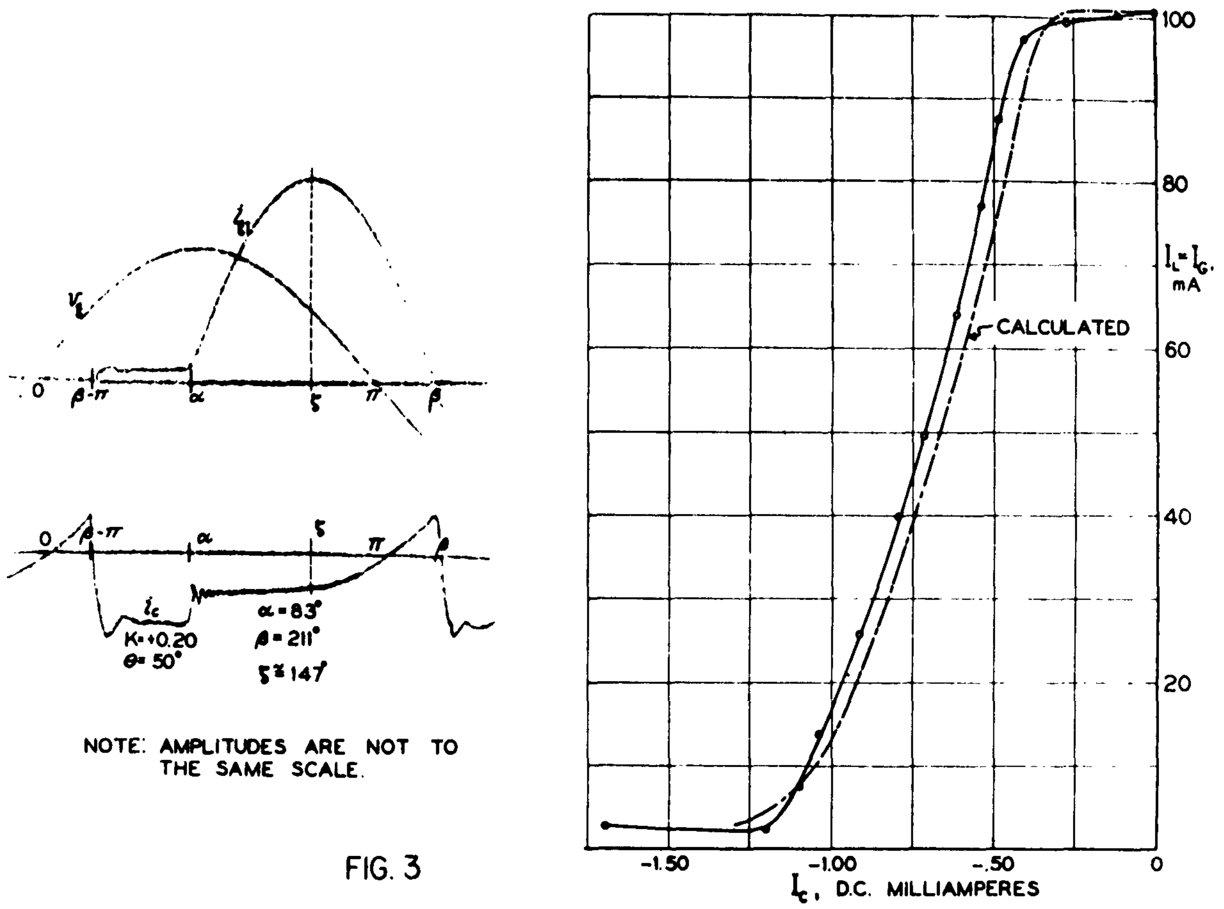
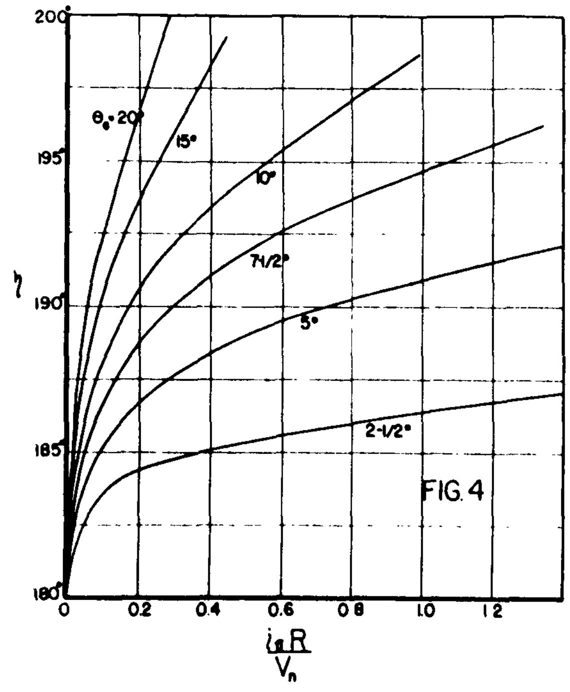


FIG. 3

Fig. 3 A.C. output. Non-rectified inductive load.

Fig. 4 The extinction angle of gate currents; rectified loads with discharge paths.



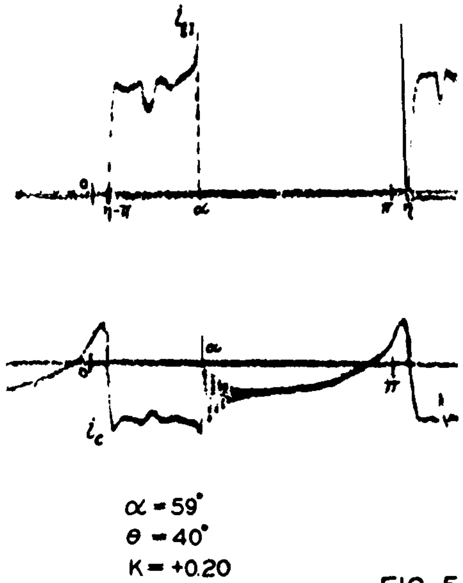


FIG. 5

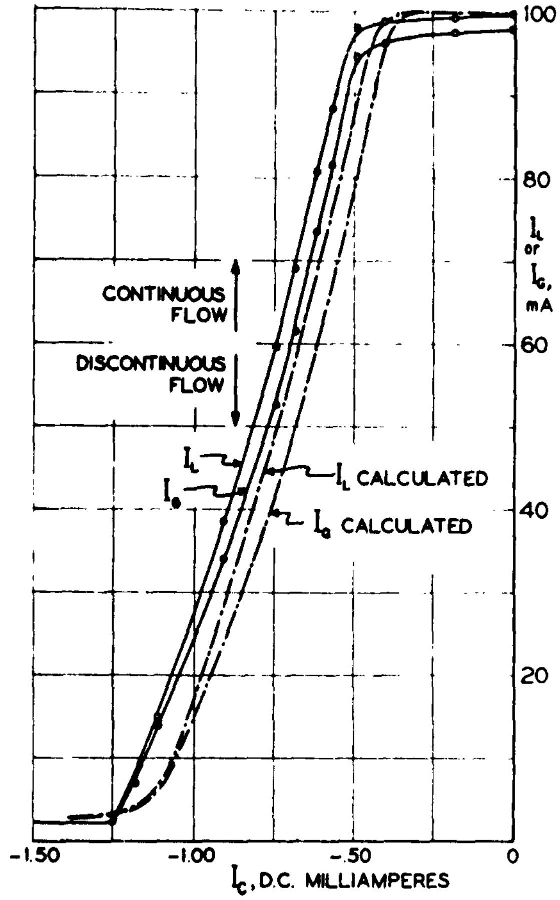


Fig. 5 Rectified inductive load with discharge paths.